Optimal decentralized valley-filling charging strategy for electric vehicles

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A B S T R A C T
Uncoordinated charging load of electric vehicles (EVs) increases the peak load of the power grid, thereby increasing the cost of electricity generation. The valley-filling charging scenario offers a cheaper alternative. This study proposes a novel decentralized valley-filling charging strategy, in which a day-ahead pricing scheme is designed by solving a minimum-cost optimization problem. The pricing scheme can be broadcasted to EV owners, and the individual charging behaviors can be indirectly coordinated. EV owners respond to the pricing scheme by autonomously optimizing their individual charge patterns. This device-level response induces a valley-filling effect in the grid at the system level. The proposed strategy offers three advantages: coordination (by the valley-filling effect), practicality (no requirement for a bidirectional communication/control network between the grid and EV owners), and autonomy (user control of EV charge patterns). The proposed strategy is validated in simulations of typical scenarios in Beijing, China. According to the results, the strategy (1) effectively achieves the valley-filling charging effect at 28% less generation cost than the uncoordinated charging strategy, (2) is robust to several potential affecters of the valley-filling effect, such as (system-level) inaccurate parameter estimation and (device-level) response capability and willingness (which cause less than 2% deviation in the minimal generation cost), and (3) is compatible with device-level multi-objective charging optimization algorithms.

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1. Introduction

As alternatives to conventional vehicles, plug-in hybrid electric vehicles (PHEVs) and battery electric vehicles (BEVs) are expected to reduce petroleum-based energy consumption and greenhouse gas emissions. Consequently, their use is likely to expand drastically in the near future [1]. According to China’s “Twelfth Five Year Plan”, the number of EVs (PHEVs and BEVs) in Beijing will likely exceed 100,000 by 2017 [2].

As EVs comprise an increasing percentage of the vehicle population, unregulated charging behaviors (especially immediate charging after arriving home in the evening) will impact the power grid in two ways: (1) by increasing the peak load and local marginal prices (LMPs) and (2) by decreasing system reliability [5]. The control of EVs’ charging loads, which is also essential to smart grids, becomes a feasible option for this issue [3]. The three potential ways of participating in a smart grid are load shifting, regulation, and spinning reserve [4].

Load shifting or valley-filling charging strategies have been widely reported. These studies focus on shifting the controllable EV load to less congested hours compared with that in an unregulated charging strategy. Valley-filling charging algorithms can be categorized into two broad classes on the basis of their EV charge patterns: centralized approaches [6–10] and decentralized approaches [11–13].

In a centralized approach, an imaginary centralized controller communicates with each EV, collects necessary information, solves specific optimization problems, and directly controls individual charging patterns. To reduce the dimensionality in the corresponding optimization problem, engineers frequently introduce aggregators between the power grid and EV owners, forming a decentralized hierarchical scheme [2,7,9,10]. Developing the computer/communication/control network and incentive program to implement this conceptual framework presents a major challenge [8]. Thus, centralized approaches are usually considered as long-term solutions to valley filling.

A decentralized approach offers more realistic alternative. In these approaches, EV owners determine individual charge patterns on the basis of an electricity pricing scheme or non-price incentives. The grid power demand can be altered by adjusting the electricity price, typically by a dual-tariff scheme [14]. While this time-varying pricing scheme encourages EV owners to charge their vehicles during valley hours, it introduces an undesirable second peak in the electricity load [3]. Zhongjing et al. [12] constructed
a non-cooperative game strategy, in which each EV pursues a cost-minimizing charge pattern based on dynamic pricing schemes iteratively. This strategy drives a unique Nash equilibrium that results in the valley-filling charging effect. Although the algorithm allows autonomous charging behaviors, the iteration process requires a high-reliability bi-directional communication network between the power grid and EVs. Thus, similar to the abovementioned centralized strategies, this strategy is unsuitable for short-term applications. Ahn et al. [11] proposed an optimal decentralized charging strategy, in which the distributed EV chargers receive a simple command from the centralized grid controller, and thereby determine their individual charge patterns. This strategy behaves as the global optimal solution of a linear optimization problem, achieving near-optimal valley-filling effect. Such a command-based strategy requires no complex bi-directional network, but its valley-filling effect depends on the willingness of EV owners to relinquish charging autonomy for device-level objectives. The main challenge in such a decentralized algorithm is how to balance system-level with device-level objectives, which often constitutes a trade-off problem [3].

The system-level objective of the overnight charging scenario is to realize a valley-filling charging effect that reduces peak load and overall generation cost. At the device level, this charging strategy are usually to optimize the individual charging patterns to minimize electricity cost and battery degradation [16]. Sikha et al. developed a varying current decay (VCD) charge pattern for a lithium ion cell. VCD enables faster charging and lower capacity fade than conventional constant current/constant voltage (CC–CV) charge patterns [17]. To maximize the useful lifetime of lithium ion cells, Rahimian et al. optimized charging currents as a function of cycle number [22,23]. Liu et al. adopted the Taguchi method in an optimization technique for designing rapid charge patterns [24]. Applying the power loss model to the battery and charger, Wang proposed that electricity cost could be minimized by efficiency-optimized charge patterns [18]. Benedikt et al. investigated the trade-off between two competitive optimizations: cost and battery lifespan [19]. Adopting a first-principles electrochemistry based battery model [25], Bashash et al. optimized the PHEV charge pattern in terms of battery longevity and energy cost [16]. They used a multi-objective genetic optimization algorithm to balance the two conflicting device-level objectives. The resulting Pareto fronts of this algorithm described the trade-off between energy cost and battery health. Although these abovementioned device-level charging algorithms generate different “optimal” charge patterns, they possess one common feature: they require charging autonomy at the device level.

An ideal valley-filling charging algorithm satisfies the following three criteria.

1. **Coordination**: An optimal valley-filling effect is achieved by the aggregated charging load of all EVs connected in the grid.
2. **Practicality**: No reliance on a bidirectional communication/control network.
3. **Autonomy**: Allowing device-level charging autonomy and appropriate balance between competing objectives at two levels.

To the best of our knowledge, none of the existing valley-filling charging strategies [7–13] possess all three characteristics. This study proposes a novel decentralized optimal valley-filling charging strategy that fulfills these three criteria. The fundamental design is an optimized day-ahead two-dimensional time-power-varying pricing scheme for dynamic electricity use.

The pricing scheme is transmitted from the centralized grid broadcaster via a unidirectional communication network and is received by the intelligent chargers installed in the EV. Thereafter, the chargers determine their individual charge patterns. When EV owners optimize the varying device-level objectives, the grid automatically achieved optimized or near-optimized valley-filling charging effect. Therefore, the proposed strategy realizes multi-level optimization.

The paper is organized as follows. Section 2 describes the system and defines the optimization problem for the proposed valley-filling charging strategy. Section 3 introduces the fundamentals of the proposed strategy; namely, the pricing scheme and its design. Section 4 verifies the robustness of the proposed strategy in a series of simulations. The conclusions are presented in Section 5.

### 2. System description and problem definition

**Fig. 1** shows the schematic of the grid–EVs system for the proposed valley-filling charging strategy. In this system, the charging behaviors of an EV fleet are indirectly coordinated by a pricing scheme. The EV owners are expected to optimize their individual charge patterns in response to the pricing scheme, enabling the aggregated charging load $P_{EV,agg}(t)$ to achieve a valley-filling effect.

The parameters of the grid–EV system are discussed in Sections 2.1 and 2.2. Using Beijing as an example, the valley-filling charging optimization problem is defined in Section 2.3. A preliminary analysis of the problem is provided in Section 2.4. The detailed pricing scheme based on the optimization problem is proposed in the next chapter.

#### 2.1. Grid demand and generation cost

**Fig. 2** shows the variation of typical grid power demand with time of day in Beijing. Data were collected at 15-min intervals during the summer and winter of 2011. This study uses the summer profile as an example, assuming that an accurate day-ahead forecast of non-EV power load $P_{nonEV}(t)$ is available [11].

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**Fig. 1.** Schematic of proposed grid–EVs system.
The electricity generation cost is usually defined as a function of generation power:

$$C = C(P),$$

where $P$ is the instantaneous power generated and the generation cost $C(P)$ is expressed in $$/h.

In Fig. 2, the areas below 6500 MW, between 6500 MW and 11,600 MW, and above 11,600 MW are defined as the base, intermediate, and peak loads, respectively [15]. The base load is usually supplied from large coal-fired power plants, which are expensive to build but relatively cheap to operate. Other power plants with reverse characteristics, such as gas turbines, supply the intermediate and peak loads. Using the method proposed in [15], we construct the piecewise linear electricity generation cost curve, as shown in Fig. 3. Electricity generation cost models involve modeling many generation characteristics and controls, including fuel costs, economic dispatch and unit commitment [27]. In this paper, we adopt a simple steady-state generation cost model and make the assumption that cost functions are convex [21]. In general, the convexity can usually be guaranteed by dispatching plants in order of their operating costs from lowest to highest [15].

In this study, the convex electricity generation cost function is generated by fitting the piecewise curve, as shown in Fig. 3. The magnitudes and detailed shape of the convex curve do not reflect the true cost of electricity in Beijing. Without considering the ramping cost and explicitly employing the techniques of unit commitment and economic dispatch [26], the resulting flat val-

![Fig. 3. Piecewise linear and electricity generation cost curves C(P).](image)

The optimization objective of the power grid seeks to minimize the overall generation cost during a period of time $[t_0, t_n]$ by direct
or indirect control of EV charge patterns \( P_{EV}(t,n) \). The optimization problem is defined as follows:

\[
\min_{P_{EV}(t,n)} \int_{t_0}^{t_f} C(P(t)) \cdot dt
\]

(4)

where

\[
P(t) = P_{nonEV}(t) + P_{EVagg}(t).
\]

(5)

\[
P_{EVagg}(t) = \sum_{n=1}^{N} P_{EV}(t,n).
\]

(6)

Here \( P(t) \) is the total grid load, comprising the non-EV load \( P_{nonEV}(t) \) and aggregated EV load \( P_{EVagg}(t) \) contributed by all EVs during \( t \in [t_0, t_n] \). For every \( n \), \( P_{EV}(t,n) \) is the charge pattern of \( EV(n) \). \( P_{EVagg}(t) \) is the sum of all \( P_{EV}(t,n) \).

Two constraints are imposed on this optimization problem. First, we impose the charging performance constraint (CPC), which is described as

\[
\text{constraint I:} \int_{t_0}^{t_f} P_{EV}(t,n) \cdot dt = D(n), n = 1, 2, \ldots, N
\]

(7)

CPC requires that all EVs are charged overnight to the upper limit of their battery capacity \( SOC_{max} \).

The second constraint is the charge flexibility constraint (CFC), which primarily embodies the factors that limit power withdrawal from the grid [5]. This constraint is described as follows:

\[
\text{constraint II:} 0 \leq P_{EV}(t,n) \leq P_{EV, max}(t,n) \cdot \delta(t,n), n = 1, 2, \ldots, N
\]

(8)

where

\[
P_{EV, max}(t,n) = \min(P_{charger, max}(n), P_{battery, max}(t,n)).
\]

(9)

\[
\delta(t,n) = \begin{cases} 1, \text{when EV \#n is plug-in} \\ 0, \text{when EV \#n is plug-off} \end{cases}
\]

(10)

CFC incorporates the maximal charging power constraint \( P_{max}(t,n) \) and EV plug-state constraint \( \delta(n) \). \( P_{max}(t,n) \) is maximal charging power of the charger or battery (denoted \( P_{charger, max}(n) \) and \( P_{battery, max}(t,n) \), respectively), whichever is the smaller.

\( P_{charger, max}(n) \) is usually constant for each EV, whereas \( P_{battery, max}(t,n) \) depends upon the battery status. In existing designs, \( P_{battery, max}(t,n) \) usually exceeds \( P_{charger, max}(n) \), which deactivates the maximal charging power constraint of the battery. Thus, \( P_{max}(t,n) \) is limited solely by \( P_{charger, max}(n) \).

Regarding CFC, in few cases, the non-EV load valley at night may be so deep that it remains unfilled, even if all EVs charge at their maximum charging power [5,13]. An analysis of this situation is beyond the scope of the present study.

2.4. Preliminary analysis of the optimization problem

In the optimization problem described by Eq.(4), the aggregated control variable \( P_{EVagg}(t) \) comprises the control variables \( P_{EV}(t,n) \).

First, we analyze the optimal aggregated control variable \( P_{EVagg}(t) \).

Under CFC, the total energy demanded by the EV load \( Energy_{EV, total} \) is estimated from the parameters in Table 1.

\[
Energy_{EV, total} = (SOC_{max}(n) - E(SOC_{plug-in}(n))) \cdot E(C(n)) \cdot N
\]

(12)

Here \( N \) is the assumed number of EVs in Beijing, and \( E(C(n)) \) and \( E(SOC_{plug-in}(n)) \) are the mathematical expectations of battery capacity and initial SOC when plugged in, respectively, whose values are listed in Table 1.

Next, we allocate \( Energy_{EV, total} \) to a fixed non-EV load profiles \( P_{nonEV}(t) \). Because the electricity generation cost \( C(P) \) is inherently convex, the optimal solution \( f_{EV agg}(t) \) must achieve a perfect (completely flat) valley-filling effect, as shown in Fig. 5. The proof is provided in Appendix A. This effect is analogous to a valley (power) being filled with water (energy). The optimal solution \( P_{EV, agg}(t) \) is expressed as follows (for derivation, see Appendix A).

\[
P_{EV, agg}(t) = \max\{0, P_{valley-filling} - P_{nonEV}(t)\}, t \in [t_0, t_m]
\]

(13)

where \( P_{valley-filling} \) defined in Eq. (A.7) of Appendix A, is a constant describing the “water” height of the filled valley. For a single valley, the optimal solution \( P_{EV, agg}(t) \) can be reformulated as:

\[
P_{EV, agg}(t) = \begin{cases} P_{valley-filling} - P_{nonEV}(t), t \in [t_{valley-start}, t_{valley-end}] \\ 0, \text{else} \end{cases}
\]

(14)

where \( t_{valley-start} \) and \( t_{valley-end} \) are the start and end times of the single valley, respectively (see Fig. 5). From this figure, we observe that the load valley in Beijing can support as much as 42% EV penetration before requiring extra investment in electricity infrastructure. In the case of 20% EV penetration, 7725 MW of energy “water” is poured into the load valley, and the valley is perfectly filled from \( t_{valley-start} = 00:15 \) to \( t_{valley-end} = 07:15 \) at a height of \( P_{valley-filling} = 8250 \) MW.

In general, the optimal solution \( P_{EV, agg}(t) \) is unique, but it may permit multiple combinations of \( P_{EV}(t,n) \) [13]. In centralized approaches [7,8,10] and some decentralized approaches [11], the
centralized controller computes a specific combination of \( P_{EV}(t, n) \), and requires each EV to determine its individual charge pattern following the direct command \( P_{EV}(t, n) \). In the remainder of this study, we seek an alternative approach that guides EV owners toward autonomously determining their individual charge patterns \( P_{EV}(t, n) \) while forming the aggregated optimal solution \( P_{EV_{agg}}(t) \) as well.

### 3. Design of pricing scheme to achieve valley filling

The overall aim of the proposed charging strategy is to optimize \( P_{EV_{agg}}(t) \) by indirectly controlling the charging behaviors of EV owners \( P_{EV}(t, n) \) in accordance with a pricing scheme. The design process of the pricing scheme is summarized in Fig. 6. To analyze how EV owners adapt their charging behaviors to a general pricing scheme, we solve an optimization problem in Section 3.1. In Section 3.2, a specific pricing scheme with undetermined coefficients \( K(t) \) is established and the corresponding individual charge patterns \( P_{EV}(t, n) \) are expressed as functions of \( K(t) \). In Section 3.3, \( K(t) \) values are determined such that the individual charge patterns \( P_{EV}(t, n) \) optimize the aggregated solution \( P_{EV_{agg}}(t) \). The key formulas relevant to each section are listed in the right panels of Fig. 6.

#### 3.1. Response of charging behaviors to electricity pricing scheme

In the power industry, smart pricing has long been considered as a methodology for demand side management (DSM) [30]. Popular smart pricing options include real-time pricing, time-of-use pricing, dual-tariff pricing, and critical-peak pricing [20,21]. Apart from these time-varying pricing schemes, the power-varying pricing scheme has been widely discussed and several proposals exist [15]. In such a "(power) demand charge," consumers are billed for both energy (kWh) and power (kW) usage. In this study, a novel smart pricing scheme that varies in both time and power forms the core of the proposed valley-filling charging strategy.

This time-power-varying pricing scheme for the valley period is constructed as follows:

\[
\text{price} = \text{price}(t, P), t \in [t_{\text{valley-start}}, t_{\text{valley-end}}].
\]

(15)

This pricing scheme consists of several convex discrete price-\( P \) curves (red lines in Fig. 7). The time step is given as \( \Delta t \). The convexity of price-\( P \) curves is defined as

\[
\frac{d^2 \text{price}(t, P)}{dP^2} \geq 0, t \in [t_{\text{valley-start}}, t_{\text{valley-end}}],
\]

(16)

where price is expressed in $/h. This curve describes the increase in electricity price per unit with charging power \( P \).

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**Section 3.1** Charging behaviors in response to a general pricing scheme

**Section 3.2** Charging behaviors in response to a specific pricing scheme with undetermined coefficients \( K(t) \)

**Section 3.3** Find specific \( K(t) \) so that individual charge patterns \( P_{EV}(t, n) \) can form \( P_{EV_{Agg}}(t) \)

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**Fig. 6.** Design process for the optimal pricing scheme.

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#### 3.2. Charging behaviors in response to a specific pricing scheme with undetermined coefficients \( K(t) \)

Device-level optimal charging algorithms are usually implemented by an imaginary intelligent charger [16–18] that automatically optimizes the charge pattern. The intelligent charger engages small electricity consumers in optimizing their individual charge patterns in response to the pricing scheme.

The optimization objectives may vary considerably among EV owners. First, we consider the simplest case, in which all EV owners aim to minimize their individual overall electricity cost \( F_{EV}(n) \). The optimization problem faced by \( EV(n) \) is then formulated as follows:

\[
\min_{t_{\text{start}}, n} F_{EV}(n) = \int_{t_{\text{start}}, n}^{t_{\text{end}}, n} \text{price}(t, P_{EV}(t, n)) \cdot dt
\]

(17)

subject to constraint: \( \int_{t_{\text{start}}, n}^{t_{\text{end}}, n} P_{EV}(t, n) \cdot dt = D(n) \)

(18)

where

\[
\begin{align*}
&\{ t_{\text{start}}, n = \max(t_{\text{plug-in}}(n), t_{\text{valley-start}}) \\
&\{ t_{\text{end}}, n = \min(t_{\text{plug-off}}(n), t_{\text{valley-end}})
\end{align*}
\]

(19)

Here \( t_{\text{start}}, n \) and \( t_{\text{end}}, n \) are the times at which \( EV(n) \) begins and terminates the charging process, respectively.

Eqs. (18) and (19) describe CPC and the plug-in state constraint of the CFC for \( EV(n) \), respectively. This optimization problem ignores the constraint of maximal charging power. The impact of this constraint on the aggregated charging load will be investigated in Section 4.2.

According to the CAERC survey, more than 99% of the vehicles have been plugged in before \( t_{\text{valley-start}} = 00:15 \) [28]. Thus, we assume that all EVs have been plugged in before \( t_{\text{valley-start}} \).

\[
\begin{align*}
&t_{\text{start}}, n = \max(t_{\text{plug-in}}(n), t_{\text{valley-start}}) \\
&t_{\text{end}}, n = \min(t_{\text{plug-off}}(n), t_{\text{valley-end}})
\end{align*}
\]

(20)

The charge pattern for \( EV(n) \) that minimizes the electricity cost \( P_{EV}(t, n) \) satisfies the following "equivalent price slope condition" (see Appendix B for deviation):

\[
\frac{d\text{price}(t, P_{EV}(t, n))}{dP_{EV}(t, n)} = \frac{d\text{price}(t, P_{EV}(t, n))}{dP_{EV}(t, n)} \cdot t_i, t_j
\]

\( \in [t_{\text{start}}, n], t_{\text{end}}, n] \).

(21)

Two charge patterns that satisfy Eq. (21) are illustrated in Fig. 7. All charging points \( P_{EV}(t, n) \) for \( EV(n) \) satisfy the "equivalent price slope condition," and the optimal solution \( P_{EV}(t, n) \) is unique (see Appendix B for proof).

---

**Fig. 7.** Response of optimal charging behaviors to the time-power-varying pricing scheme.
Eq. (21) provides a potential means of indirectly controlling the charge patterns \( P_{\text{EV}}(t, n) \) to form the aggregated optimal solution \( P_{\text{EV-agg}}'(t) \) in the time–power-varying pricing scheme.

### 3.2. Construction of the pricing scheme

Eq. (21) relates the optimal charging patterns \( P_{\text{EV}}(t, n) \) to the pricing scheme \( \text{price}(t, P) \). In this section, the detailed pricing scheme \( \text{price}(t, P) \) is designed so that the charge patterns \( P_{\text{EV}}'(t, n) \) can form the optimal solution \( P_{\text{EV-agg}}'(t) \).

As shown in Appendix B, if each price-\( P \) curve is convex, the optimal charging pattern for EV\((n)\) \( P_{\text{EV}}'(t, n) \) is unique. Given that \( C(P) \) is convex, \( \text{price}(t, P) \) is formulated as a series of perspectives of \( C(P) \).

\[
\text{price}(t, P) = K(t)C\left( \frac{P}{K(t)} \right), \quad t \in [t_{\text{valley-start}}, t_{\text{valley-end}}]
\]  

(22)

Here \( K(t) \) are time-varying undetermined coefficients. In Eq. (22), each price-\( P \) curve is generated by scaling \( C(P) \) by a fixed aspect ratio. Fig. 8 illustrates three price-\( P \) curves at 02:00, 04:00, and 06:00. These plots do not necessarily imply a relationship between pricing function \( \text{price}(t, P) \) and electricity cost function \( C(P) \). The basic convex function \( C(P) \) can be replaced by any suitable one-dimensional convex curve.

To study the charging behaviors of all EVs in response to the pricing scheme, the specific pricing scheme Eq. (22) is substituted into the “equivalent price slope condition” of Eq. (21) to yield

\[
\frac{dC\left( \frac{P_{\text{EV}}'(t, n)}{K(t)} \right)}{d\left( \frac{P_{\text{EV}}'(t, n)}{K(t)} \right)} = \frac{1}{f(t)}, \quad t, t_j \in [t_{\text{n-start}}, t_{\text{n-end}}], t \neq t_j.
\]

(23)

Because the function \( C(P) \) is convex, the point on \( C(P) \) whose tangent satisfies Eq. (23) is unique. Thus,

\[
P_{\text{EV}}'(t, n) = \frac{P_{\text{EV}}'(t, n)}{K(t)} = \frac{P_{\text{EV}}'(t, j)}{K(t_j)} = \frac{P_{\text{EV}}'(t, n)}{K(t_j)}, \quad t, t_j \in [t_{\text{n-start}}, t_{\text{n-end}}], t \neq t_j.
\]

(24)

where the closed interval \( I_{[t_{\text{n-start}}, t_{\text{n-end}}]} \) defined by Eq. (19) is a subset of \( I_{\text{valley-start, valley-end}} \).

Eq. (24), deduced from the “equivalent price slope condition” of Eq. (21), describes the charging behaviors of all EVs in response to the specific pricing scheme of Eq. (22).

In terms of Eq. (24), the optimal charge pattern \( P_{\text{EV}}'(t, n) \) for EV\((n)\) is formulated as follows.

\[
P_{\text{EV}}'(t, n) = \begin{cases} \frac{\text{D}(t, n)}{I_{t_{\text{n-end}}} \left( K(t) \right)} \frac{1}{t_{\text{n-end}}} \left( K(t) \right) dt \quad t \in [t_{\text{n-start}}, t_{\text{n-end}}] \\ 0 \quad \text{else} \end{cases}
\]

(25)

Equivalently, it can also be expressed as follows.

\[
P_{\text{EV}}'(t, n) = \begin{cases} \frac{\text{D}(t, n)}{I_{t_{\text{valley-end}}} \left( K(t) \right) dt \frac{1}{t_{\text{valley-end}}} \left( K(t) \right) \text{valley-end}}} \quad t \in [t_{\text{valley-start}}, t_{\text{valley-end}}] \\ 0 \quad \text{else} \end{cases}
\]

(26)

The undetermined coefficients \( K(t) \) are then determined so that the device-level optimal charge patterns \( P_{\text{EV}}'(t, n) \) can contribute to forming the system-level optimal charging load \( P_{\text{EV-agg}}'(t) \).

### 3.3. Optimal \( K(t) \)

In this study, the optimal \( K(t) \) is found by exploration. This section progressively introduces three \( K(t) \) strategies and compares their valley-filling effects.

Intuitively, the undetermined coefficients \( K(t) \) should follow the trend of the optimal aggregated EV load \( P_{\text{EV-agg}}'(t) \). In this way, each EV charges at a higher rate when the load valley is deeper. Thus, we propose the first \( K(t) \) strategy as

\[
K_1(t) = \frac{P_{\text{EV-agg}}'(t)}{P_{\text{valley-agg}}} \cdot N
\]

(27)

where \( N \) is the number of EVs in Beijing.

The effectiveness of the proposed \( K(t) \) is assessed by MATLAB simulation, in which each EV autonomously optimizes its individual charge pattern \( P_{\text{EV}}'(t, n) \) according to Eq. (26). The simulation results are shown in Fig. 9(a) and (b), where the result of \( K_1(t) \) is plotted in red. Because some EVs depart from home earlier than \( t_{\text{valley-end}} \) and are charged at higher than expected power level, the pricing scheme (Eq. (22)) with \( K_1(t) \) cannot achieve a perfect valley-filling effect, unless all EVs remain plugged in until \( t_{\text{valley-end}} \).

As shown in Fig. 9(d), the number of plugged-in EVs \( N_{\text{EV}}(t) \) begins decreasing around 5 am. The red line in Fig. 9(a) and (b) lies above the optimal valley-filling curve before 5 am and is lower thereafter.

To understand the formation of such imperfect valley-filling effects, we construct an illustrative scenario involving four EVs. Under the \( K_1(t) \) pricing strategy, the individual charging patterns of the EVs should follow the same trend throughout the load valley period \( [t_{\text{valley-start}}, t_{\text{valley-end}}] \) as shown in Fig. 10(a). However, because EV(1) leaves earlier than \( t_{\text{valley-end}} \), it must increase its charge rate during the first three time slots. The result is imperfect valley-filling charging, as shown in Fig. 10(b).

Ahn et al. [11] proposed an updatable equivalent \( K(t) \) strategy (unrelated to pricing scheme), whose valley-filling results are almost identical to those of \( K_1(t) \).

Correcting \( K_1(t) \) by the number of plugged-in EVs \( N_{\text{EV}}(t) \) (Fig. 9(d)), the second \( K(t) \) strategy is proposed as follows:

\[
K_2(t) = \frac{P_{\text{EV-agg}}'(t)}{P_{\text{valley-agg}}} \cdot N_{\text{EV}}(t)
\]

(28)

The outcome of this strategy is plotted in green in Fig. 9(a) and (b). Under the pricing scheme with corrected \( K_2(t) \), the valley-filling effect is much improved and meets the requirements of practical implementation. However, we remain focused on finding the theoretical optimal \( K(t) \), if it exists.

To this end, we introduce the \( K_3(t) \) strategy. This strategy is derived in Appendix C. Fig. 10(c) illustrates the basic construction of \( K_3(t) \); the charge patterns for \( N_{\text{EV}}(t) \) plugged-in EVs at time \( t \) are determined from the updated load valley trajectory, generated by removing the consumption of early-leave \( (N-N_{\text{EV}}(t)) \) EVs from the previous load valley trajectory. The theoretical optimal \( K_3(t) \) is recursively formulated as follows (see Appendix C for deviation).
Fig. 9. Valley-filling effect, simulated with different $K(t)$: (a) Normal view, (b) enlarged view, and (c) repeated simulation. (d) The number of plugged EVs, simulated with different seeds of random generator.

Fig. 10. Results of implementing the pricing strategy with different $K(t)$, on an illustrative EV fleet of 4 vehicles. (a) $K_1(t)$ with constant $N_{EV}(t)$, (b) $K_1(t)$ with varying $N_{EV}(t)$, (c) $K_3(t)$ with varying $N_{EV}(t)$, and (d) comparison of different $K(t)$ values.
The simulation result of $K_3(t)$ pricing scheme is plotted in blue in Fig. 9(a) and (b). The valley-filling effect under this strategy is superior to that of both $K_2(t)$ and $K_3(t)$ pricing schemes. Small fluctuations arise because $D(n)$ is assumed as a time-static random variable independent of $\tau_{plug-off}(n)$ (see Eq. (C.4) in Appendix C). Repeating the simulation with different random generator seeds, small fluctuations persist, as shown in Fig. 9(c). In future research, these small fluctuations could be further suppressed by determining the correlation function between $\tau_{plug-off}(n)$ and $D(n)$.

Finally, we specify a constant high-electricity unit price outside of the load valley hour. The final pricing scheme is then formulated as follows:

$$\text{price}(t, P) = \begin{cases} 
K_3(t)C_{EV,\text{agg}} \frac{P_{EV,\text{agg}}(t)}{\tau_{\text{valley-filling}}}, & t \in [t_{\text{valley-start}}, t_{\text{valley-end}}] \\
3 \times C_{EV,\text{agg}} \frac{P_{EV,\text{agg}}(t)}{\tau_{\text{valley-filling}}}, & t \notin [t_{\text{valley-start}}, t_{\text{valley-end}}].
\end{cases}$$  

(30)

The valley-filling effect in Fig. 9 exemplifies the coordination characteristic of the proposed strategy. To design the pricing scheme, the information from the individual EV to grid is not required.

In this section, the impacts of underestimated and overestimated connected EV numbers $N$ and corresponding $\text{Energy}_{EV,\text{total}}$ are investigated in scenarios 1 and 2, respectively. In practice, charging efficiency of battery/charger system is less than 1 and varies with charging power [6,9]. The paper assume a unitary charging efficiency implicitly in Eqs. (3) and (12), which can also result in a similar underestimated $\text{Energy}_{EV,\text{total}}$ scenario. Scenario 3 studies the impact of non-EV load profiles $P_{\text{nonEV}}(t)$ that are delayed by one hour.

The simulation results of these three scenarios are shown in Fig. 12. Fig. 12(a1) and (a2) plots the simulation results of Scenario 1, in which the connected EVs are underestimated by 40% ($N_{\text{under}} = 0.6N$). The actual required charging energy $\text{Energy}_{EV,\text{total}}$ is 66% higher than the energy available in the shortened estimated valley hours (00:45–06:45). Compared with the perfect valley-filling effect (the blue dashed line in Fig. 12(a2)), a part of the EV charging load is shifted from both sides to the center of the valley, implying a higher generation cost. Fig. 12(b1) and (b2) plots the simulation results of scenario 2, in which the connected EVs are overestimated by 33% ($N_{\text{over}} = 1.33N$). In this case, the actual required charging energy $\text{Energy}_{EV,\text{total}}$ is 25% lower than the energy available in the extended estimated valley hours (00:00–07:30). Compared with the perfect valley-filling (the blue dashed line in Fig. 12(b2)), a part of the EV load is moved from the center to both sides of the valley, signifying a higher generation cost. The simulation results of scenario 3 are plotted in Fig. 12(c1 and c2). In this scenario, the estimated non-EV load profiles $P_{\text{nonEV}}(t)$ are delayed by one hour. The resulting pricing scheme prompts consumers to delay their charging to beyond the optimal timing period, causing imperfect valley-filling.

In the enlarged views of the three scenarios (Fig. 12(a2, b2, and c2), the deviations from perfect valley-filling effect are clearly observed. However, in the normal views (Fig. 12(a1, b1, and c1), the valley-filling is revealed as acceptable for practical implementation.
To quantitatively evaluate the deviation from perfect valley-filling, we define the generation cost of the EV load $C_{EV}$ as

$$C_{EV} = \int_{T_{start}}^{T_{end}} C(P(t)) \cdot dt \cdot \left( \text{Energy}_{EV, total} \right)$$

$$= \int_{T_{start}}^{T_{end}} P(t) \cdot dt$$

where $P(t)$ is the total grid load defined by Eq. (5), $T_{start}$ is the time at which the first EV begins to charge, and $T_{end}$ is the time at which the last EV completes its charging. In the proposed valley-filling charging strategies, we set $T_{start} = T_{valley-start}$ and $T_{end} = T_{valley-end}$. The definition of $C_{EV}$, exemplified by the dual-tariff scenario, is illustrated in Fig. 13. First, the total cost of generation is integrated from $T_{start}$ to $T_{end}$ (the first entry of Eq. (31)), and then, it is multiplied by the energy proportion of the EV load $E_{Energy, EV, total}$ in the total energy consumption (the second entry of Eq. (31)).

The second column of Table 2 lists $C_{EV}$ under different scenarios. The cost of generating EV load ($C_{EV}$) is slightly higher ($\approx 2\%$) when the parameters are incorrectly estimated than when they are accurate, but all costs are much lower (around 28%) than in the baseline scenario. Thus, under inaccurate estimation, the valley-filling effects remain acceptable.
The long-term grid responses to these inaccurate estimations are further investigated. The long-term EV charging behavior is driven by profit, which is defined as

$$\text{Profit} = F_{\text{total}} - C_{\text{EV}},$$

where the total user fee $F_{\text{total}}$ is the sum of the overall charging electricity costs $F_{\text{EV}}(n)$ across all EVs:

$$F_{\text{total}} = \sum_{n=1}^{N} F_{\text{EV}}(n) = \sum_{n=1}^{N} \left( \int_{t_{\text{start}}}^{t_{\text{end}}} price(t, P_{\text{EV}}(t, n)) \cdot dt \right)$$

The results are shown in the third and fourth columns of Table 2. Regardless of which parameters are incorrectly estimated, the grid loses profit in sub-optimal scenarios. Thus, the grid is spontaneously inspired to improve the accuracy of estimates in long-term period.

### 4.2. Device-level impact of response capability

The capability of EV owners to respond to the pricing scheme is largely governed by the maximal charging power $P_{\text{max}}(t, n)$ defined in Eq. (9). Enhancing the response capacity is equivalent to relaxing CFC or increasing the maximal charging power of the charger.

Note that the constraint of maximal charging power $P_{\text{EV,max}}(t, n)$ is in the optimization problem defined by Eqs. (17)-(19). Because of limited response capability, the optimal individual charge pattern $P_{\text{EV}}(t, n)$ may not be thoroughly evaluated for all EVs. In this section, we discuss the impact of this constraint on the valley-filling effect and investigate the long-term behaviors of EV owners with respect to response capability.

In most cases, the energy decay $D(n)$ is fairly small and the plug-off time $t_{\text{plug-off}}(n)$ is relatively late. However, if an EV is “busy,” i.e., its energy consumption is large, $D(x)$ and early $t_{\text{plug-off}}(x)$ may not be adequately supported by the charger or battery. In this case, EV(x) must reformulate its optimal charge pattern to minimize its total electricity cost, on the basis of the original optimal charge pattern of Eq. (26). The corrected version of the optimal charge pattern is obtained by reformulating Eq. (26) as follows:

$$P_{\text{EV}}(t, n) = \begin{cases} \frac{P_{\text{max}}(t, n)}{\Delta t} \sum_{k=t_{\text{start}}}^{t_{\text{end}}} price(t, P_{\text{EV}}(t, n))^K(t), t \in \{T_{m1}, T_{m2}, \ldots T_{mq}\} \\ 0, \text{else} \end{cases}$$

where $T_{m1}, T_{m2}, \ldots T_{mq}$ are the $q$ time slots in which the charging power $P_{\text{EV}}(t, n)$ computed by Eq. (26) exceeds the maximal charging power $P_{\text{EV,max}}(t, n)$. Eq. (34) is iterated until the corrected $P_{\text{EV}}(t, n)$ no longer exceeds $P_{\text{EV,max}}(t, n)$.

The impact of constraining the maximum charging power is evaluated in four simulations, where $P_{\text{EV,max}}(t, n) = 1.44$ kW (Level I charger), 2 kW, 2.5 kW, and 3 kW. In each case, the EVs autonomously determine their individual charge patterns according to Eq. (34). The simulation results are illustrated in Fig. 14. In this figure, imposing stricter constraints on the maximal charging power amplifies the fluctuations in the total grid load curve $P(t)$ during the valley hours. When the maximal charging power is 3 kW, the fluctuations are very slight (the blue dashed line in Fig. 14). Considering that most PEVs are limited to 3.3 kW by the onboard power converter [5], the response capability exerts negligible effect on valley filling.

Having verified that the proposed algorithm is robust to response capability, we further investigate the long-term effects of the system- and device-level behaviors on this response capability.

At the system level, the power grid is concerned with how limiting response capability impacts its profit. The results for the aforementioned four cases are calculated by Eq. (31)-(32)-(33) in Table 3. The generation cost increases as the constraints on maximal charging power tighten. However, because the total user fee $F_{\text{total}}$ increases more rapidly, the grid profit increases with stricter constraints on the maximal charging power. Thus, we obtain

---

**Table 2**

<table>
<thead>
<tr>
<th>Statistical parameter</th>
<th>EV load cost/10^5$</th>
<th>User fee/10^5$</th>
<th>Profit/10^5$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Baseline</td>
<td>3.9110</td>
<td>--</td>
<td>--</td>
</tr>
<tr>
<td>Dual-tariff</td>
<td>2.8075</td>
<td>3.0662</td>
<td>0.3139</td>
</tr>
<tr>
<td>Correct estimate</td>
<td>2.7522</td>
<td>2.7568</td>
<td>2.7525</td>
</tr>
<tr>
<td>Correct estimate</td>
<td>2.7586</td>
<td>2.7531</td>
<td>2.7537</td>
</tr>
<tr>
<td>Over estimate</td>
<td>2.7599</td>
<td>2.7573</td>
<td>2.7548</td>
</tr>
<tr>
<td>Offset 1 h</td>
<td>2.7840</td>
<td>3.0562</td>
<td>3.0662</td>
</tr>
</tbody>
</table>

* The high ramping cost caused by undesirable second load peak is not included.

**Table 3**

<table>
<thead>
<tr>
<th>$P_{\text{max}}$</th>
<th>EV load cost/10^5$</th>
<th>User fee/10^5$</th>
<th>Profit/10^5$</th>
</tr>
</thead>
<tbody>
<tr>
<td>No limit</td>
<td>2.7522</td>
<td>3.0662</td>
<td>0.3139</td>
</tr>
<tr>
<td>3 kW</td>
<td>2.7525</td>
<td>3.0873</td>
<td>0.3384</td>
</tr>
<tr>
<td>2.5 kW</td>
<td>2.7531</td>
<td>3.2083</td>
<td>0.4552</td>
</tr>
<tr>
<td>2 kW</td>
<td>2.7537</td>
<td>3.3937</td>
<td>0.6400</td>
</tr>
<tr>
<td>1.44 kW</td>
<td>2.7548</td>
<td>3.7737</td>
<td>1.0189</td>
</tr>
</tbody>
</table>

---

**Fig. 14.** Impact of imposing a maximal EV charging power on valley filling: (a) enlarged view and (b) normal view.
\[ \text{Profit}_{\text{total}} \leq \text{Profit}_{<2.5 \text{ kW}} \leq \text{Profit}_{2.5 \text{ kW}} \leq \text{Profit}_{>4.4 \text{ kW}}. \]  
(35)

In other words, the extra generation cost incurred by limited response capability is completely covered by device-level EV owners. Therefore, an incentive program that increases device-level response capability is not required by the grid.

At the device level, we select four EVs and investigate their long-term behaviors. The overall electricity cost \( F_{\text{EV}}(n) \) of EV(n) is related to the maximal charging power \( P_{\text{EV,\text{max}}}(t, n) \) and plug-off time \( t_{\text{plug-off}}(n) \), as illustrated in Fig. 15. The overall electricity cost of EV(n), \( F_{\text{EV}}(n) \), decreases as the maximal charging power \( P_{\text{EV,\text{max}}}(t, n) \) increases (Fig. 15(a)), and it also decreases as the plug-off time \( t_{\text{plug-off}}(n) \) becomes later (Fig. 15(b)). Therefore, each EV owner is inherently encouraged to upgrade his/her charger and extend the plug-in time. The long-term behaviors of such spontaneous improvement provide valuable insight for future smart grid applications.

Spontaneous upgrade behaviors are more efficient than grid-launched incentive programs for upgrading chargers because each EV owner can autonomously decide whether an upgrade is necessary by assessing his/her personal situation. For example, if the constraint of maximal charging power is deactivated in EV(999), the charger of that vehicle requires no upgrade (see Fig. 15(a)). However, by upgrading his/her charger, the “busy” owner of EV(282) can reduce his/her electricity cost \( F_{\text{EV}}(n) \) by 50%. The effectiveness of the proposed charging algorithm also depends on the willingness of EV owners to respond to the proposed pricing scheme developed in Section 3. The above discussion assumed that all EV owners seek to solely minimize their overall electricity cost. In other words, the EV owners exert the highest possible degree of response willingness.

In practice, the response willingness of EV owners is usually weaker than the ideal case. Owners may be insensitive to overall electricity cost \( F_{\text{EV}}(n) \) or may prioritize other objectives (such as preserving battery integrity). If the charging efficiency of battery/charger system is less than 1 [6,9], the owners minimize their overall electricity cost by optimizing net charge patterns and minimizing charging losses simultaneously. Thus, the impact of non-ideal charging efficiency can also be regarded as weaker response willingness. As discussed in Section 1, device-level optimal charging objectives are personal and the resulting optimal charge patterns are distinguishable. Among the various device-level charging algorithms, the multi-objective optimizing algorithm proposed in [16], which introduces a series of Pareto fronts, promises to coordinate multiple conflicting objectives at the device level. In this section, to investigate the impact of weakened response willingness, the algorithm of [15] is integrated into the proposed valley-filling charging algorithm.

Algorithms that achieve multi-objective charging optimization in specific battery models are worth exploring but beyond the scope of this study. Here we focus on the impact of response willingness on the proposed valley-filling charging strategy, with no attempt to develop a multi-objective optimization algorithm. Therefore, the trade-off between electricity cost and battery degradation is demonstrated as a series of directly generated Pareto fronts (see Table 4), which is similar to the Pareto fronts developed in [16]. For further details, see [16].

In general, battery degradation increases with the charging rate. Here the battery degradation, quantified by the added film resistance \( R \), is assumed to become more severe at higher maximal charging power constraint \( P_{\text{EV,\text{max}}}(t, n) \), as shown in Table 4. Note that Fig. 15(a) in Section 4.2 relates the overall electricity cost of EV(n), \( F_{\text{EV}}(n) \), to the maximal charging power constraint \( P_{\text{EV,\text{max}}}(t, n) \). Thus, when constructing the three Pareto fronts, \( P_{\text{EV,\text{max}}}(t, n) \) is regarded as the intermediate variable between \( F_{\text{EV}}(n) \) and \( R \).

Instead of solving the single-objective optimization problem of Eq. (17), each EV owner personalizes his/her charge pattern by comparing the Pareto fronts constructed from his/her individual preference [16]. This comparison process is equivalent to solving Eq. (36) rather than Eq. (17).

\[ \min_{F_{\text{EV}}(n)} J_{\text{EV}}(n,x) \]
(36)

Here \( J_{\text{EV}}(n,x) \) is the total cost function of EV(n) for Pareto front x, calculated by

\[ J_{\text{EV}}(n,x) = F_{\text{EV}}(n,x) + J_{\text{B}}(n,x), \]
(37)

where \( J_{\text{B}}(n,x) \) is the overall electricity cost of EV(n) for Pareto front x, and \( J_{\text{B}}(n,x) \) is the cost of battery degradation caused by the added film assistance \( R \).

The total cost function \( J_{\text{B}}(n,x) \) incorporates many parameters, including energy decency \( D(n) \), plug-off time \( T_{\text{plug-off}}(n) \), battery characteristics, and user preference. Therefore, \( J_{\text{B}}(n,x) \) may differ markedly from one EV to another. In the proposed charging algorithm, each EV can independently select its optimal Pareto front \( x'(n) \).

### Table 4
Illustrative Pareto fronts of EV(282).

<table>
<thead>
<tr>
<th>Pareto front</th>
<th>( P_{\text{EV,\text{max}}} ) (kW)</th>
<th>( F_{\text{EV}}/$ )</th>
<th>Added film resistance ( R/10^{-3} \Omega )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>3</td>
<td>0.7877</td>
<td>1.77</td>
</tr>
<tr>
<td>2</td>
<td>2</td>
<td>0.8314</td>
<td>0.97</td>
</tr>
<tr>
<td>3</td>
<td>1.44</td>
<td>1.6444</td>
<td>0.77</td>
</tr>
</tbody>
</table>

**Fig. 15.** Relationships between the overall electricity cost \( F_{\text{EV}} \) and (a) maximal charging power \( P_{\text{EV,\text{max}}} \) and (b) plug-off time \( T_{\text{plug-off}} \).
\[ J_{EV}(n, x'(n)) \leq J_{EV}(n, x), n = 1, 2, \ldots N, x = 1, 2, 3 \quad (38) \]

For simplicity, we assume that all EVs select the same Pareto front \( x' \).

\[ x'(n) = x', n = 1, 2, \ldots N \quad (39) \]

Under this assumption, all EVs are subject to the same constraint of maximal charging power constraint \( P_{EV,max}(t, n) \). Equivalent scenarios have been investigated in Section 4.2. Imposing a maximal charging power constraint raises the profit of the grid (see Eq. (35)); i.e.,

\[ \text{Profit}_{\text{max}} \geq \text{Profit}_{\text{unlimit}}. \quad (40) \]

Table 5

<table>
<thead>
<tr>
<th>Strategy</th>
<th>Coordination</th>
<th>Practicality</th>
<th>Autonomy</th>
<th>EV load cost/10^5$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Unregulated (baseline)</td>
<td>×</td>
<td>✓</td>
<td>✓</td>
<td>3.9110</td>
</tr>
<tr>
<td>Dual-tariff</td>
<td>×</td>
<td>✓</td>
<td>✓</td>
<td>2.8075</td>
</tr>
<tr>
<td>Command-based ((K_i(t))) [11]</td>
<td>✓</td>
<td>✓</td>
<td>×</td>
<td>2.7534</td>
</tr>
<tr>
<td>Nash equilibrium [12,21]</td>
<td>✓</td>
<td>×</td>
<td>×</td>
<td>2.7519</td>
</tr>
<tr>
<td>Centralized strategies [10]</td>
<td>✓</td>
<td>×</td>
<td>×</td>
<td>2.7519</td>
</tr>
<tr>
<td>The proposed strategy ((K^p(t)))</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>2.7522</td>
</tr>
</tbody>
</table>

* The high ramping cost caused by undesirable second load peak is not included.
* This value will be 2.7522 if the proposed \( K_i(t) \) is applied to the command-based strategy of [11].
* Assume that the perfect valley-filling effect can be achieved.

The performance of the proposed strategy was evaluated on typical situations encountered in Beijing. The simulations incorporated likely affecters of the valley-filling effect; namely, inaccurate estimation of system-level parameters and lower response capability and weaker response willingness at the device level. The effectiveness and robustness of the proposed charging strategy was verified by the results. The proposed charging strategy can coordinate the charging behaviors of EV owners and achieve near-perfect valley-filling effect under different scenarios, reducing the cost of EV load generation by 28%.

To summarize this example, limited response willingness leads to imperfect valley filling (see Fig. 14). However, by accounting for personalized optimization objectives, we can realize better results than the scenario when we assume the highest possible response willingness (Eqs. (38) and (40)). Therefore, the proposed valley-filling charging strategy can balance system-level and device-level objectives.

The pricing scheme transmitted from grid to EV is not an either-or command. The EV users can choose the degree of response to the pricing scheme. Thus, the EV owners’ charging autonomy, which is a prerequisite for device-level charging patterns optimization, is guaranteed without disproportionately affecting the valley filling effect.

To emphasize the advantages of the proposed charging strategy, we compare it with other charging scenarios/strategies by examining the characteristics of coordination, practicality and autonomy, as shown in Table 5. All the existing valley-filling charging strategies [10–12,21] have the characteristic of coordination, but none of them possesses both practicality and autonomy. To the best of our knowledge, the proposed charging strategy is the first one that possesses all three characteristics.

5. Conclusions

This study presents a decentralized valley-filling strategy that optimizes vehicle charging in a grid–EV system. In this proposed strategy, the power grid indirectly coordinates the charging behaviors of all EVs by implementing a time-power-varying pricing scheme. When the EV owners individually optimize their charging patterns in response to the pricing scheme (device-level optimization), the grid automatically achieves a valley-filling charging effect (system-level optimization).

This pricing scheme is based on two convex optimization problems, detailed in Appendices A and B. The first (inequality-constrained problem) determines the optimal aggregated EV load profiles \( P_{EV,agg}(t) \). The second (equality-constrained problem) establishes the optimal charging behaviors \( P_{EV}(t, n) \) in response to a pricing scheme with undetermined coefficients \( K_i(t) \) at the device level. The optimal \( K^p(t) \), which guarantees that the charge patterns \( P_{EV}(t, n) \) across all EVs form \( P_{EV,agg}(t) \), is solved by another optimization problem, presented in Appendix C.

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Appendix A

The time-discretized version of the convex optimization formulated by Eq. (4) is

\[ \min_{P_{EV,agg}(t)} \sum_{i=0}^{m} C_i (\text{Profit}_{\text{moniV}(t_i)} + P_{EV,agg}(t_i)) \cdot \Delta t \quad (A.1) \]

subject to: \[ \begin{align*}
\sum_{i=1}^{m} P_{EV,agg}(t_i) \cdot \Delta t - \text{Energy}_{EV,\text{total}} &= 0 \\
-P_{EV,agg}(t_i) \cdot \Delta t &\leq 0, i = 1, 2, \ldots, m
\end{align*} \quad (A.2) \]
The Lagrangian of this time-discretized form is given as follows:

\[
L(P_{EV,agg}(t), \lambda, \nu) = \sum_{i=1}^{m} C(P_{nonEV}(t) + P_{EV,agg}(t)) \cdot \Delta t \\
- \sum_{i=1}^{m} \lambda_i P_{EV,agg}(t) \cdot \Delta t + \nu \\
\cdot \left( \sum_{i=1}^{m} P_{EV,agg}(t) \Delta t - \text{Energy}_{EV,\text{total}} \right)
\] (A.3)

while the Karush–Kuhn–Tucker (KKT) conditions are

\[
\begin{aligned}
\nabla_{P_{EV,agg}} L(P_{EV,agg}(t), \lambda^*, \nu^*) &= 0, i = 1, 2, \ldots, m \\
\lambda_i^* &\geq 0, i = 1, 2, \ldots, m \\
-P_{EV,agg}(t) \cdot \Delta t &\leq 0, i = 1, 2, \ldots, m \\
-\lambda_i^* \cdot P_{EV,agg}(t) \cdot \Delta t &\leq 0, i = 1, 2, \ldots, m \\
\sum_{i=1}^{m} P_{EV,agg}(t) \Delta t - \text{Energy}_{EV,\text{total}} &= 0
\end{aligned}
\] (A.4)

Eliminating the slack variable \( \lambda^* \), Eq. (A.4) reduces to

\[
\left( \frac{dC(P_{nonEV}(t) + P_{EV,agg}(t))}{dP_{EV,agg}(t)} + \nu^* \right) \cdot P_{EV,agg}(t) = 0, i = 1, 2, \ldots, m
\] (A.5)

whose solution is

\[
P_{EV,agg}(t) = \max \{0, (C)^{-1}(-\nu^*) - P_{nonEV}(t)\}, i = 1, 2, \ldots, m.
\] (A.6)

Here \((C)^{-1}\) is the inverse function of the derivative of \(C(P)\). Because \(C(P)\) is guaranteed convex, the derivative function \(C\) monotonically increases; therefore, its inverse function is validly expressed as \((C)^{-1}\).

Thus, \(P_{\text{valley-filling}}\) which first appears in Eq. (14), is defined as

\[
P_{\text{valley-filling}} = (C)^{-1}(-\nu^*).
\] (A.7)

Substituting Eq. (A.6) in the fifth line of Eq. (A.4), we obtain

\[
\sum_{i=1}^{m} \max \{0, P_{\text{valley-filling}} - P_{nonEV}(t)\} \cdot \Delta t - \text{Energy}_{EV,\text{total}} = 0
\] (A.8)

which is solved from Eq. (17) and (18) is

\[
L(P_{EV}(t, n), \nu) = \sum_{i=0}^{m} \text{price}(t_i, P_{EV}(t_i, n)) \cdot \Delta t \\
+ \nu \cdot \left( \sum_{i=0}^{m} P_{EV}(t_i, n) \cdot \Delta t - N_{EV}(t) \right)
\] (B.1)

where \(t = t_0, t_1, \ldots, t_m, t_0 = t_{\text{start}, \text{agg}}\) and \(t_m = t_{\text{end}, \text{agg}}\).

The optimality condition is formulated as

\[
\nabla_{P_{EV}(t, n)} L(P_{EV}(t, n), \nu^*) = 0, i = 1, 2, \ldots, m.
\] (B.2)

which reduces to the “equivalent price slope condition.”
References


